Algebra 3/Trigonometry – Summer Work

Welcome upcoming Algebra III/Trigonometry student! This packet was designed for you to practice the mathematical skills that all students are expected to have upon entering this course. Take the time to complete all the problems and make sure you understand how to do the problems. These prerequisite skills will make your time in Algebra III/Trigonometry much easier and will greatly increase the likelihood of a successful school year**. If you are unsure of how to complete some of the problems, please click on the links provided to access online videos for extra help. You may also search the topics listed below at www.Khanacademy.org for additional resources.**

The Algebra 3/Trigonometry Summer Packet will be due **the first week of school**. This packet will not be graded. However, you will be quizzed on the material from the packet during the beginning weeks of school.

Prerequisite Skills for Algebra 3/Trigonometry

1) Simplifying Algebraic Expressions , Multiplying Polynomial Expressions
2) Factoring Polynomial Expressions
   - Greatest Common Factor (GCF)
   - Difference of Squares
   - Factoring Quadratic into two Binomials
   - Factoring by Grouping
   - Factor by the sum/difference of cubes
3) Simplifying integer exponents , Scientific Notation
4) Simplifying Radical Expressions
5) Solving Linear Equations
6) Linear Equations
   - Graphing Linear Equations
   - Writing Linear Equations
   - Writing Equations of Parallel Lines
   - Writing Equations of Perpendicular Lines
7) Solving Absolute Value Equations
8) Solving and Graphing Inequalities
9) Solving Linear Systems
   - Substitution Method
   - Elimination Method
10) Pythagorean Theorem

Section 1: Simplifying Expressions

Simplify the following expressions by adding, subtracting, multiplying, and dividing where necessary.

1. $4x(x-7)-8x^2$ Distribute

$4x^2-28x-8x^2$ CLT

$\boxed{-4x^2-28x}$

2. $(x^2+4x+5)-(3x-3)$ Distribute

$x^2+4x+5-3x+9$ CLT

$\boxed{x^2+x+14}$

3. $(3x+1)(2x+5)$ FOIL

$6x^2+15x+2x+5$ CLT

$\boxed{6x^2+17x+5}$

4. $(2x-3)(x^2+x+1)$

$2x^3+2x^2+2x-3x^2-3x-3$

$\boxed{2x^3-x^2-x-3}$

Section 2: Factoring Polynomials

Completely factor the following polynomials. Using the method specified.

Factor out the Greatest Common Factor.

5. $3x^2-18x$ GCF = 3x

$\frac{3x}{3x}\quad\frac{18x}{3x}$

$=\boxed{3x(x-6)}$

6. $-5rs-15r^2s+10rt$ GCF = -5r

$\frac{-5rs}{-5r}\quad\frac{15r^2s}{-5r}\quad\frac{10rt}{-5r}$

$=\boxed{-5r(s+3rs-2t)}$

Difference of Squares If the expression is not factorable, write not factorable.

7. x^2-36

$=\boxed{(x+6)(x-6)}$

8. $4x^2-25$

$=\boxed{(2x+5)(2x-5)}$

9. x^2+81

Not Factorable

Factor each into two binomials (remember to check for a GCF first).

10. $x^2+7x+10$

$=\boxed{(x+5)(x+2)}$

11. $x^2+10x+25$

$=(x+5)(x+5)$

$=\boxed{(x+5)^2}$

12. $4x^2-8x-32$ GCF = 4

$\frac{4x^2}{4}\quad\frac{8x}{4}\quad\frac{32}{4}$

$=4(x^2-2x-8)$

$=\boxed{4(x-4)(x+2)}$

13. $3x^2-11x-4$

$=\boxed{(3x+1)(x-4)}$

Factor by Grouping.

14. $(3x^3-3x^2)+(2x-2)$

$=3x^2(x-1)+2(x-1)$

$=\boxed{(x-1)(3x^2+2)}$

15. $(4x^3+16x^2)-(x-4)$

$=4x^2(x+4)-1(x+4)$

$=\boxed{(x+4)(4x^2-1)}$

Factor using the sum/difference of cubes formula. $8 = (2)^3$, $27 = (3)^3$ $(a^3 + b^3) = (a+b)(a^2 - ab + b^2)$

16. $x^3 - 64$ $64 = (4)^3$

$= (x-4)(x^2+4x+16)$

17. $8x^3 + 27$ $(a^3 - b^3) = (a-b)(a^2 + ab + b^2)$

$= (2x+3)(4x^2-6x+9)$

Section 3: Integer Exponents

Simplify each expression using the Law of Exponents, express the answer so all exponents are positive.

18. $x^0y^{-2} = \frac{1}{y^2}$

19. $(8x^3)^2 = 8^2x^6$

$= 64x^6$

20. $-4x^{-1} = \frac{-4}{x}$

21. $\frac{x^{-2}y^3}{xy^4} = \frac{1}{x^3y}$

22. $\left(\frac{3x}{4y^2}\right)^{-2} = \left(\frac{4y^2}{3x}\right)^2$

$= \frac{4^2y^4}{3^2x^2} = \frac{16y^4}{9x^2}$

Write numbers out that are written in scientific notation.

23. 3.86×10^7 move decimal 7 places to the right

$= 38{,}600{,}000$

24. 1.23×10^{-5} move decimal 5 places to the left

$= .0000123$

Section 4: Simplifying Radicals

Simplify each expression.

25. $\sqrt{8} = \sqrt{4 \cdot 2}$

$= 2\sqrt{2}$

26. $\sqrt{50} = \sqrt{25 \cdot 2}$

$= 5\sqrt{2}$

26. $\sqrt[3]{-64} = \sqrt[3]{(-4)^3}$

$= -4$

28. $\sqrt[3]{16} = \sqrt[3]{8 \cdot 2}$

$= 2\sqrt[3]{2}$

29. $-5\sqrt{32} = -5\sqrt{16 \cdot 2}$

$= -5 \cdot 4\sqrt{2}$

$= -20\sqrt{2}$

30. $\sqrt{80x^4}$

$= \sqrt{16 \cdot 5 \cdot x^4}$

$= 4x^2\sqrt{5}$

31. $\sqrt{125x^3}$

$= \sqrt{25 \cdot 5 \cdot x^2 \cdot x}$

$= 5x\sqrt{5x}$

32. $2\sqrt{3} + 5\sqrt{3}$

$= 7\sqrt{3}$

33. $6\sqrt{12} - 4\sqrt{3}$

$= 6\sqrt{4 \cdot 3} - 4\sqrt{3}$

$= 12\sqrt{3} - 4\sqrt{3}$

$= 8\sqrt{3}$

34. $3\sqrt{3} \cdot 4\sqrt{15}$

$= 12\sqrt{45}$

$= 12\sqrt{9 \cdot 5}$

$= 12 \cdot 3\sqrt{5}$

$= 36\sqrt{5}$

Section 5: Solving Linear Equations

Solve the following equations. Check your solution.

35. $5n - 10 = 8 - 31n$

$+31n \quad +31n$

$36n - 10 = 8$

$+10 \quad +10$

$\frac{36n}{36} = \frac{18}{36}$

$n = \frac{1}{2}$

36. $5(2 - a) = 0$

$10 - 5a = 0$

$-10 \quad -10$

$\frac{-5a}{-5} = \frac{-10}{-5}$

$a = 2$

37. $4x - 8 = 2(x - 5)$

$4x - 8 = 2x - 10$

$-2x \quad +8 \quad -2x \quad +8$

$\frac{2x}{2} = \frac{-2}{2}$ $\quad x = -1$

38. $\frac{x}{6} = \frac{9}{2}$

$\frac{2x}{2} = \frac{54}{2}$ $\quad x = 27$

39. $8x - (3x + 2) = 3x - 10$

$8x - 3x - 2 = 3x - 10$

$5x - 2 = 3x - 10$

$-3x \quad +2 \quad -3x \quad +2$

$\frac{2x}{2} = \frac{-8}{2}$ $\quad x = -4$

40. $x(2x - 3) = (2x + 1)(x - 4)$ FOIL

$2x^2 - 3x = 2x^2 - 8x + x - 4$

$2x^2 - 3x = 2x^2 - 7x - 4$

$-2x^2 \quad -2x^2$

$-3x = -7x - 4$

$+7x \quad +7x$

$\frac{4x}{4} = \frac{-4}{4}$ $\quad x = -1$

Section 6: Writing and graphing linear equations.

Slope Formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope-Intercept Form: $y = mx + b$ Point-Slope Form: $y - y_1 = m(x - x_1)$

Write the slope intercept form of the equation of each line given the slope and the y-intercept.

41. Slope $= 2$, y-intercept $= -2$

$y = 2x - 2$

42. Slope $= -\frac{3}{5}$, y-intercept $= 2$

$y = -\frac{3}{5}x + 2$

Write the slope-intercept form of the equation of the line that passes through the given point with the given slope. start with point-slope form

43. Through: $(-3, 5)$, slope$= -1$ $\quad (x_1, y_1)$ $\quad m$

$y - 5 = -1(x + 3)$

$y - 5 = -x - 3$

$+5 \quad +5$

$y = -x + 2$

44. Through: $(5, 0)$, slope$= -\frac{3}{5}$ $\quad (x_1, y_1)$ $\quad m$

$y - 0 = -\frac{3}{5}(x - 5)$

$y = -\frac{3}{5}x + 3$

Write the slope-intercept form of the equation of the line that passes through the given points.

45. Through: $(-2, -4)$ and $(-1, 4)$ $\quad (x_1, y_1)$

$m = \frac{4+4}{-1+2}$

$m = \frac{8}{1}$

$m = 8$

$y - 4 = 8(x + 1)$

$y - 4 = 8x + 8$

$+4 \quad +4$

$y = 8x + 12$

46. Through: $(0, -2)$ and $(4, 4)$ $\quad (x_1, y_1)$

$m = \frac{4+2}{4-0}$

$m = \frac{6}{4}$

$m = \frac{3}{2}$

$y + 2 = \frac{3}{2}(x - 0)$

$y + 2 = \frac{3}{2}x - 2$

-2

$y = \frac{3}{2}x - 2$

47. Write the equation of the line parallel to $y = 3x + 2$ that passes through $(-1, -2)$.

$m = 3$ $\quad (x_1, y_1)$

$y + 2 = 3(x + 1)$

$y + 2 = 3x + 3$

$-2 \quad -2$

$y = 3x + 1$

48. Write the equation of a line perpendicular to $y = \frac{1}{4}x - 5$ that passes through $(1, 1)$.

(x_1, y_1)

$m = -4$

$y - 1 = -4(x - 1)$

$y - 1 = -4x + 4$

$+1 \quad +1$

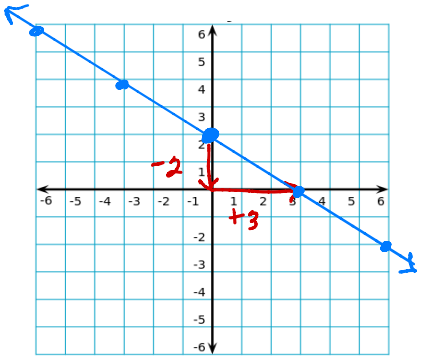
$\boxed{y = -4x + 5}$

Graph the equation.

49. $y = -\frac{2}{3}x + 2$

$m = -\frac{2}{3}$

$y\text{-int} = 2$



50. $12 - 4y = -4x$

$12 - 4y = -4x + 12$

$+12$

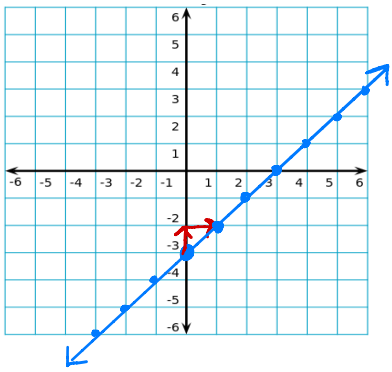
$-4y = -4x + 12$

$\frac{-4y}{-4} = \frac{-4x}{-4} + \frac{12}{-4}$

$y = 1x - 3$

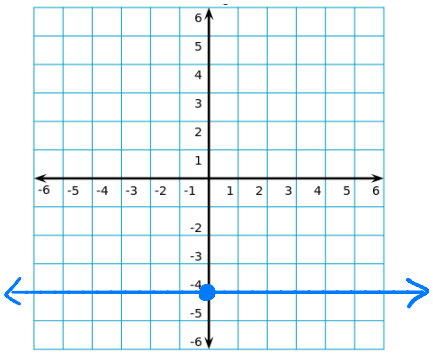
$m = 1$

$y\text{-int} = -3$



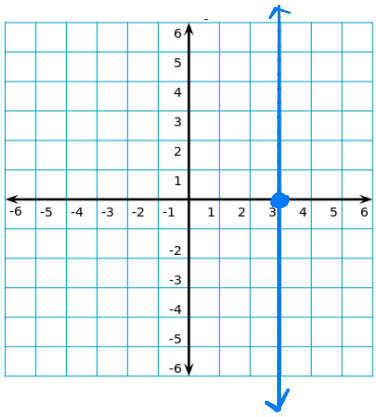
51. $y = -4$

horizontal line thru y-axis at $(0, -4)$



52. $x = 3$

vertical line thru x-axis at $(3, 0)$



Section 7: Solving Inequalities and Graphing the Solution.

Solve each inequality and graph its solution.

53. $a + 8 < 5$

$-8 \quad -8$

$a < -3$

54. $-11 \leq n - 8$

$+8 \quad +8$

$-3 \leq n$

Rewrite as $n \geq -3$

55. $-13m < 39$

$\frac{-13m}{-13} < \frac{39}{-13}$ Flip inequality symbol

$m > -3$

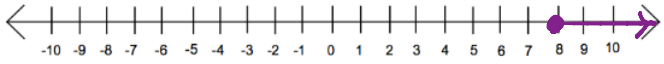
56. $-8(r+3) < -88$

$-8r - 24 < -88$

$+24 \quad +24$

$\frac{-8r}{-8} < \frac{-64}{-8}$ flip inequality symbol

$r > 8$



Section 8: Solve Absolute Value Equations

57. $|2d - 5| = 13$

$2d - 5 = 13$ $\quad +5 \quad +5$

$\frac{2d}{2} = \frac{18}{2}$

$d = 9$

$2d - 5 = -13$ $\quad +5 \quad +5$

$\frac{2d}{2} = \frac{-8}{2}$

$d = -4$

58. $|5 + 2x| = 9$

$5 + 2x = 9$ $\quad -5 \quad -5$

$\frac{2x}{2} = \frac{4}{2}$

$x = 2$

$5 + 2x = -9$ $\quad -5 \quad -5$

$\frac{2x}{2} = \frac{-14}{2}$

$x = -7$

Section 9: Solving Linear Systems

Solve the following systems of equations by <u>substitution</u>.

59. $x + y = 7$

$x = y + 9$

CLT $y + 9 + y = 7$

$2y + 9 = 7$

$-9 \quad -9$

$\frac{2y}{2} = \frac{-2}{2}$

$y = -1$

$x = y + 9$

$x = -1 + 9$

$x = 8$

Solution: $(8, -1)$

60. $y = 2x + 32$

$2x + y = 60$

CLT $2x + 2x + 32 = 60$

$4x + 32 = 60$

$-32 \quad -32$

$\frac{4x}{4} = \frac{28}{4}$

$x = 7$

$y = 2x + 32$

$y = 2(7) + 32$

$y = 14 + 32$

$y = 46$

Solution: $(7, 46)$

Solve the following systems of equations by <u>elimination</u>.

61. $x + 2y = 10$

$8x - 2y = 8$

$9x = 18$

$\frac{9x}{9} = \frac{18}{9}$

$x = 2$

$x + 2y = 10$

$2 + 2y = 10$

$-2 \quad -2$

$\frac{2y}{2} = \frac{8}{2}$

$y = 4$

Solution: $(2, 4)$

62. $-3x + 2y = 14$

$3(x - y = -3) \rightarrow 3x - 3y = -9$

$-3x + 2y = 14$

$-y = 5$

$\frac{-y}{-1} = \frac{5}{-1}$

$y = -5$

$x - y = -3$

$x + 5 = -3$

$-5 \quad -5$

$x = -8$

Solution: $(-8, -5)$

Section 10 – Geometry Review

Find the missing side of the following right triangles using Pythagorean's Theorem.

63. $a = 2,\ b = 2,\ c = ?$

$a^2 + b^2 = c^2$

$2^2 + 2^2 = c^2$

$4 + 4 = c^2$

$\sqrt{c^2} = \sqrt{8}$

$c = \pm 2\sqrt{2}$

$c = +2\sqrt{2}$

64. $a = 8,\ c = 10,\ b = ?$

$a^2 + b^2 = c^2$

$8^2 + b^2 = 10^2$

$64 + b^2 = 100$

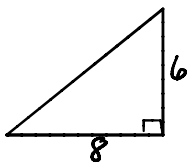
$b^2 = 100 - 64$

$\sqrt{b^2} = \sqrt{36}$

$b = \pm 6$

$b = +6$

65. Find the area of the triangle in #64.



$A = \frac{1}{2}b \cdot h$

$A = \frac{1}{2}(8)(6)$

$A = \frac{48}{2}$

Area = 24 square units